Spatial Relationships

based on Shekhar/Chawla

Operations on spatial objects in Object Model

| Set theory based | Union, Intersection, Containment, |
| Topological      | Touches, Disjoint, Overlap, etc. |
| Directional      | East, North-West, etc. |
| Metric           | Distance |

Topological Relationships

- Topological Relationships
  - invariant under elastic deformation (without tear, merge).
  - Two countries which touch each other in a planar paper map will continue to do so in spherical globe maps.
- Topology is the study of topological relationships
- Example queries with topological operations
  - What is the topological relationship between two objects A and B?
  - Find all objects which have a given topological relationship to object A?
Topological Concepts

- Interior, boundary, exterior
  - Let $A$ be an object in a "Universe" $U$.

$A$ interior of $A$ (green)
$\partial A$ boundary of $A$ (red)
$A^c$ exterior of $A$ (blue)

For curves and points:
boundary of curve: endpoints, interior: curve without endpoints

Topological Relationships in $9$-Intersection Model

Many topological relationships are captured by Egenhofer’s $9$ intersection model:

Intersections between interior, boundary, exterior of $A$, $B$:
- arranged as a $3 \times 3$ matrix

$\Gamma_9(A, B) = \begin{pmatrix}
A \cap B & A \cap \partial B & A \cap B^c \\
\partial A \cap B & \partial A \cap \partial B & \partial A \cap B^c \\
A^c \cap B & A^c \cap \partial B & A^c \cap B^c
\end{pmatrix}$

Q: How many Boolean matrices are there?

Possible Operations

Following list shows all possibilities for two connected, hole-free regions

Q: How does this model specify topological relationships between a polygon and a curve?
Modeling topological relationships in graph model