Spatial Relationships

based on Shekhar/Chawla

Operations on spatial objects in Object Model

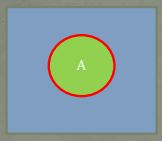
Set theory based	Union, Intersection, Containment,
Toplogical	Touches, Disjoint, Overlap, etc.
Directional	East,North-West, etc.
Metric	Distance

Topological Relationships

- Topological Relationships
  - invariant under elastic deformation (without tear, merge).
  - Two countries which touch each other in a planar paper map will continue to do so in spherical globe maps.
- Topology is the study of topological relationships
- Example queries with topological operations
  - What is the topological relationship between two objects A and B ?
  - Find all objects which have a given topological relationship to object A ?

Topological Concepts

- Interior, boundary, exterior
  - Let A be an object in a "Universe" U.



A° interior of A (green)

∂A boundary of A (red)

A^- exterior of A (blue)

For curves and points?
boundary of curve: endpoints, interior: curve without endpoints

Topological Relationships in 9-Intersection Model

Many topological relationships are captured by Egenhofer's 9 intersection model:

Intersections between
interior, boundary, exterior of A, B
A and B are spatial objects in a two dimensional plane.
-> arranged as a 3 by 3 {0,1}-matrix

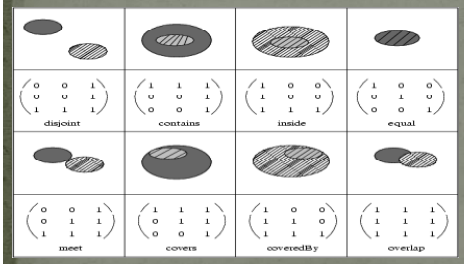
$$\Gamma_9(A,B) = \begin{pmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap B^- \\ \partial A \cap B^\circ & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^\circ & A^- \cap \partial B & A^- \cap B^- \end{pmatrix}$$

Q? How many Boolean matrices are there?

Possible Operations

Following list shows all possibilities for two connected, hole-free regions

$$\Gamma_9(A,B) = \begin{pmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap B^- \\ \partial A \cap B^\circ & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^\circ & A^- \cap \partial B & A^- \cap B^- \end{pmatrix}$$



Q: How does this model specify topological relationships between a polygon and a curve?

Modeling topological relationships in graph model